[5 PTS] Find all zeros of $f(x) = 3x^3 + 4x^2 + 5x - 6$.

ANSWER: $\frac{2}{3},\ -1 \pm \sqrt{2}\,i$

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & 4 & 5 & -6 \\ & & 2 & 4 & 6 \\ \hline & 3 & 6 & 9 & 0 \end{array}$$

$f(x) = (x - \frac{2}{3})(3x^2 + 6x + 9)$

$= 3(x - \frac{2}{3})(x^2 + 2x + 3)$

$x = \frac{-2 \pm \sqrt{4-12}}{2} = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2\sqrt{2}\,i}{2} = -1 \pm \sqrt{2}\,i$

[2 PTS] Check if $x = -2$ is a lower bound of the real zeros of $f(x) = 2x^3 + 3x^2 - 3$.
In words, give a very brief reason for your answer.

ANSWER: YES

(YES or NO)

REASON: SIGNS ALTERNATE

$\frac{1}{2}$ $\frac{1}{2}$

$$\begin{array}{r|rrrr} -2 & 2 & 3 & 0 & -3 \\ & & -4 & 2 & -4 \\ \hline & 2 & -1 & 2 & -7 \end{array}$$

[2 PTS] Use Descartes' Rule of Signs to determine the possible numbers of positive and negative zeros of $f(x) = x^5 - x^4 + 4x^3 + 15x^2 + 58x - 40$.

ANSWER:

positive zeros: 3 or 1

negative zeros: 2 or 0

$f(-x) = -x^5 - x^4 - 4x^3 + 15x^2 - 58x - 40$

ADDITIONAL QUESTIONS ON THE OTHER SIDE ➡

[4 PTS] Find a polynomial function with real coefficients that has 2 and $5+i$ as zeros. **ANSWER:** $x^3-12x^2+46x-52$

$(x-2)(x-(5+i))(x-(5-i))$

$= (x-2)((x-5)-i)((x-5)+i)$

$= (x-2)((x-5)^2-i^2)$

$= (x-2)(x^2-10x+25+1)$

$= (x-2)(x^2-10x+26)$ 2

$= x^3-10x^2+26x$
$\quad -2x^2+20x-52$

$= x^3-12x^2+46x-52$

[2 PTS] List all possible rational zeros of $f(x)=6x^5+9x^3-12x^2-24x+4$.
You do NOT need to find any actual zeros.

ANSWER: $\boxed{\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{6}}$

2

$-\frac{1}{2}$ FOR EACH ERROR OR MISSING #

$\pm \dfrac{1, 2, 4}{1, 2, 3, 6}$

[3 PTS] Write the quotient in standard form $\dfrac{3i}{(4-5i)^2}$. **ANSWER:** $-\dfrac{120}{1681} - \dfrac{27}{1681}i$

$\dfrac{3i}{16-40i+25i^2}$

$= \dfrac{3i}{-9-40i} \cdot \dfrac{-9+40i}{-9+40i}$

$= \dfrac{-27i+120i^2}{81-1600i^2} = \boxed{\dfrac{-120-27i}{1681}}$

[2 PTS] Simplify i^{75} and write in standard form. **ANSWER:** $-i$

$$\begin{array}{r} 18 \\ 4\overline{)75} \\ \underline{4} \\ 35 \\ \underline{32} \\ 3 \end{array}$$

$i^{75} = i^3 = -i$